Highly Degenerate Fermi Gases

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Introduction

Highly degenerate Fermi gases up to spin 9/2 are now realized in cold Fermi labs, therefore, here we present the study of a dilute Fermi gas. As we are dealing with a many body system, a perturbation series method will be applied. The expansion will be up to third order including any possible polarization. The universal expansion, that is up to second order, have been computed analytically. For the third order terms numerical calculations have led to the energy. The final expression obtained for the energy depends on the polarization, the gas parameter and the degeneracy, but it only stands for low density gases. Also the study of the ferromagnetic phase transition is presented.

Theory

After the application of a perturbative expansion in terms of the gas parameter (Fermi momentum times the S-wave scattering length) and the integration of each term, the final formula for the energy is:

$$\frac{E}{N}=\frac{3}{5}\varepsilon_F\left[\frac{1}{\nu}\sum_{\lambda}C_{\lambda}^{5/3}+\frac{5}{3\nu}\sum_{\lambda_1,\lambda_2}\left\{\frac{2}{3\pi}(k_Fa_0)C_{\lambda_1}C_{\lambda_2}+\frac{I_2(k_F,C_{\lambda_1},C_{\lambda_2})}{k_F^7}(k_Fa_0)^2\right.\right.$$
$$+\frac{1}{10\pi}C_{\lambda_1}C_{\lambda_2}\left(\frac{C_{\lambda_1}^{2/3}+C_{\lambda_2}^{2/3}}{2}\right)\left[(k_Fr_0)(k_Fa_0)^2+2(k_Fa_1)^3\right]\Big\}(1-\delta_{\lambda_1,\lambda_2})$$
$$\left.+\frac{5}{3}\left((\nu-1)(E_3+E_4)(k_Fa_0)^3+(\nu-1)(\nu-3)E_5(k_Fa_0)^3\right)\right]$$

Up to second order, the expression is universal as it only depends on a_0 (S-wave scattering length). Then we have higher order terms depending on r_0 (S-wave effective range) and a_1 (P-wave scattering length). Finally, three more terms are included, but E_3, E_4 and E_5 are objects computed numerically. The second order term has a function called I_2, this function is shown below.

$$I_2(k_F,C_{\lambda_1},C_{\lambda_2})=\frac{4k_F^7}{35\pi^2}C_{\lambda_1}C_{\lambda_2}\frac{C_{\lambda_1}^{1/3}+C_{\lambda_2}^{1/3}}{2}\left\{\frac{1}{4}\left(15\frac{C_{\lambda_1}^{2/3}}{C_{\lambda_2}^{2/3}}-19\frac{C_{\lambda_1}^{1/3}}{C_{\lambda_2}^{1/3}}+52-19\frac{C_{\lambda_2}^{1/3}}{C_{\lambda_1}^{1/3}}+15\frac{C_{\lambda_2}^{2/3}}{C_{\lambda_1}^{2/3}}\right)\right.$$
$$+\frac{7}{8}\left(\frac{C_{\lambda_1}}{C_{\lambda_2}}-\frac{C_{\lambda_1}^{2/3}}{C_{\lambda_2}^{2/3}}-5\frac{C_{\lambda_1}^{1/3}}{C_{\lambda_2}^{1/3}}+10-5\frac{C_{\lambda_2}^{1/3}}{C_{\lambda_1}^{1/3}}-\frac{C_{\lambda_2}^{2/3}}{C_{\lambda_1}^{2/3}}+\frac{C_{\lambda_2}}{C_{\lambda_1}}\right)\ln\left|\frac{C_{\lambda_1}^{1/3}-C_{\lambda_2}^{1/3}}{C_{\lambda_1}^{1/3}+C_{\lambda_2}^{1/3}}\right|$$
$$\left.-\frac{2C_{\lambda_1}^{1/3}}{C_{\lambda_1}^{1/3}+C_{\lambda_2}^{1/3}}\frac{C_{\lambda_1}}{C_{\lambda_2}}\ln\left|\frac{C_{\lambda_1}^{1/3}+C_{\lambda_2}^{1/3}}{C_{\lambda_1}^{1/3}}\right|-\frac{2C_{\lambda_2}^{1/3}}{C_{\lambda_1}^{1/3}+C_{\lambda_2}^{1/3}}\frac{C_{\lambda_2}}{C_{\lambda_1}}\ln\left|\frac{C_{\lambda_1}^{1/3}+C_{\lambda_2}^{1/3}}{C_{\lambda_2}^{1/3}}\right|\right\}$$

The entire expression is written in terms of C_λ, this object is the ratio between the occupation of a certain species in a general situation and the occupation in the non-polarized case. Its definition and its polarization dependence are:

$$N_\lambda\equiv C_\lambda\frac{N}{\nu}\qquad C_s=1+P(\nu-1)\quad for\quad P>0\qquad C_{-s}=1-P(\nu-1)\quad for\quad P<0$$
$$C_{\lambda\neq s}=1-P\quad for\quad P>0\qquad C_{\lambda\neq -s}=1+P\quad for\quad P<0$$

For the analysis we will use the susceptibility and the Tan's constant, defined as follows:

$$\frac{1}{\chi}=\frac{1}{n}\frac{\partial^2}{\partial P^2}\left(\frac{E}{N}\right)\qquad C=2\pi nk_F\frac{x^2}{\varepsilon_F}\frac{\partial\left(E/N\right)}{\partial x}$$

Results

For spin 1/2 the gas behaves as a non-polarized gas until a density of $k_Fa_0\approx0.85$, then a phase transition occurs, and it becomes fully polarized. If we look to all the properties, we can see that the transition is continuous. Once the gas is fully polarized the Tan's constant is zero as there is no interaction.

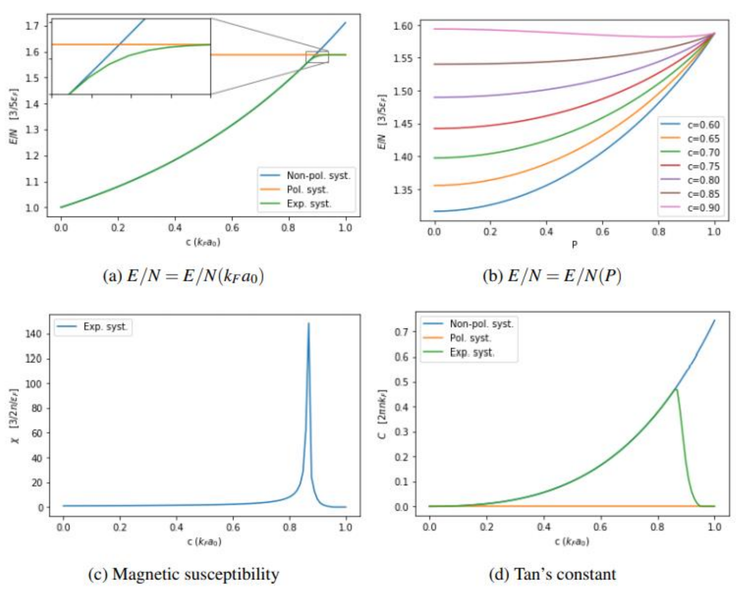

Figure 1. Representation of the energy as a function of the density and the polarization, of the magnetic susceptibility and the Tan's constant. The Fermi gas has spin 1/2. In some figures there is a line called 'Exp. syst', this curve represents the behavior of the gas. In the first figure, a zoom is done near the transition.

If we compare spin 3/2 with spin=1/2, we see the transition point has moved to a lower value. For spin 1/2 was $k_Fa_0\approx0.85$, now is $k_Fa_0\approx0.7$. Another thing to notice is the pronounced drop in the magnetic susceptibility, this is telling us that for this case a discontinuous phase transition is happening. It can also be seen in the zoom near the transition, where the change is stepped and not smooth.

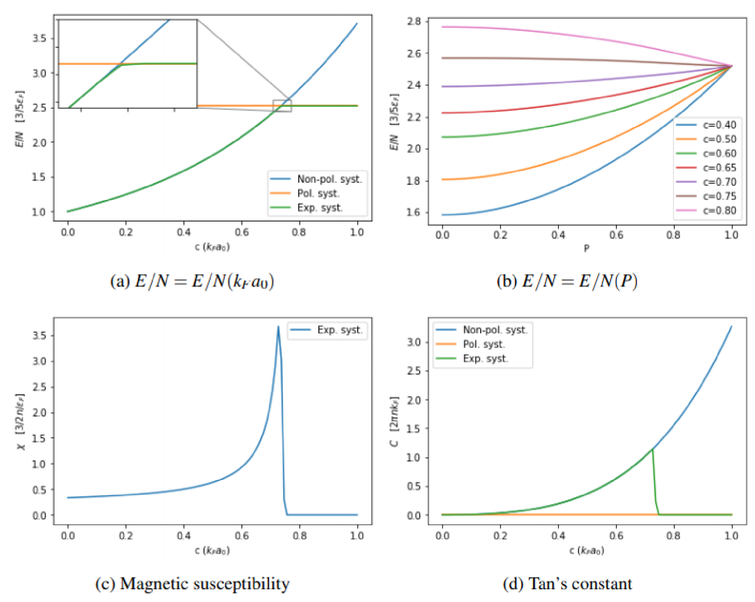

Figure 2. Representation of the energy as a function of the density and the polarization, of the magnetic susceptibility and the Tan's constant. The Fermi gas has spin 3/2. In some figures there is a line called 'Exp. syst', this curve represents the behavior of the gas. In the first figure, a zoom is done near the transition.

Polaron

The polaron is a specific case in which we have an almost fully-polarized gas. As we have an analytical expression for the energy, we can compare it with the expression of a polaron and find the chemical potential (A), the effective mass (m*) and the interaction parameter (F). The energy for the polaron is shown below, the variable x is the ratio between the residual and the dominant species.

$$E_P=\frac{3}{5}N^+\varepsilon_F^+\left(1+Ax+\frac{m}{m^*}x^{5/3}+Fx^2\right)\qquad x\equiv\frac{C_{-s}}{C_s};\quad k_F^+=k_FC_s^{1/3};\quad N^+\varepsilon_F^+=N\varepsilon_FC_s^{5/3}$$

After comparing energies, the main properties of the polaron are:

$$A=\frac{5}{3}\left[\frac{4}{3\pi}(k_F^+a_0)+\frac{2}{\pi^2}(k_F^+a_0)^2+\frac{1}{10\pi}\left(\frac{r_0}{a_0}+2\left(\frac{a_p}{a_0}\right)^3\right)(k_F^+a_0)^3\right]$$
$$m^*=\frac{m}{1-\frac{5}{3}\left[\frac{4}{5\pi^2}(k_F^+a_0)^2-\frac{1}{10\pi}\left(\frac{r_0}{a_0}+2\left(\frac{a_p}{a_0}\right)^3\right)(k_F^+a_0)^3\right]}\qquad F=\frac{5}{3}\frac{4}{3\pi^2}(k_F^+a_0)^2$$

We show the chemical potential and the effective mass as a function of the dominant density. The orange lines correspond to the expansion up to second order. We see that for the chemical potential we cannot reproduce the DMC points properly. However, if we include r_0 and a_1 we can reproduce the points almost perfectly (blue line).

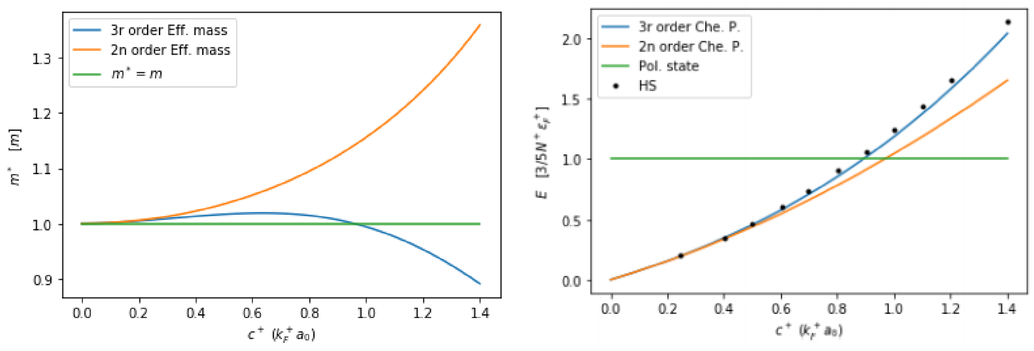

Figure 3. Representation of the chemical potential (right figure) and the effective mass (left figure) up to second order and up to second order with the effective range and the P-wave scattering length (blue line). DMC points of the chemical potential in black.

Conclusion

As a summary, we have been able of obtaining an analytical expression of the energy for any polarization up to second order (universal expansion) and also for the S-wave effective range and the P-wave scattering length. After analysing the behaviour of the Fermi gas through the energy, the magnetic susceptibility and the Tan's constant, we can say that the phase transition from paramagnetic gas to ferromagnetic gas occurs at lower a density when we increase the spin. An interesting feature though of studying the phase transition is the observation of a continuous transition for spin 1/2 while a discontinuous one happens at high degeneracy.